

# A Model to Describe Transport Properties in $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$

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## ABSTRACT

A pseudo-spin model is proposed, as a means to describe some transport properties (resistivity and Hall mobility) in  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$ . Our model is based in a double-well potential where tunneling in a given site and interaction between different lattice sites are allowed only through the excited states. Doping of the pure system by the addition of  $Pr$  increases the ratio between the activation energy and the tunneling constant. The model Hamiltonian displays some features which are present in the hydrogen-bonded ferroelectrics. Its dynamics is treated in the random phase approximation and the characteristic frequency (time) is used in a Drude formula in order to obtain some transport properties of the system, namely the electric resistivity and the Hall mobility. The quantities calculated in this work are compared with the experimental data of B. Beschoten, S. Sadewasser, G. Güntherodt and C. Quitmann [Phys. Rev. Lett.77, 1837(1996)].

Pacs number: 74.20.De, 74.25.Dw, 74.25.Fy, 74.72.Hs

## 1. Introduction

The controversy behind the mechanism which governs high temperature superconductivity (HTS) in cuprates is a subject of current interest, as can be verified in the recent literature [1,2,3].

Anderson [2] attributes the novel phenomenology present in cuprates materials to a second kind of metallic state, namely, the Luttinger liquid. Zhang [3] has proposed a SO(5) theory of cuprate superconductivity which considers that the phenomenology of these materials might be fundamentally due to a conflict between different kinds of order. Chianchi, Moretti and Piazza [4] have presented a phenomenological model for the formation of Cooper pairs in cuprate superconductors: in their description, coupling is mediated by large anharmonic oscillations along  $\hat{c}$  direction of the apical ions. On the other hand, Beschoten et al [5] have investigated the charge transport in insulating and superconducting samples of  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$ , in terms of the Hall mobility. The main motivation behind Beschoten et al work [5] was that the interplay of localization and superconductivity is of fundamental interest in the physics of electrons (holes) in strongly correlated and disordered systems such as the high-temperature superconductors (HTSC).

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\*work partially developed at DCNat - Departamento de Ciências Naturais, FUNREI - Fundação de Ensino Superior de São João del Rei. Praça Dom Helvecio, CEP:36.300-000,São João del Rei - M.G. Brazil

The aim of this paper is to propose a phenomenological model to explain the resistivity and the Hall mobility behaviors inferred from Beschoten et al data [5], performed in  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$ . The strategy of our work is first to propose a model Hamiltonian to describe the  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$  and then to study its dynamics in the random phase approximation (RPA). After doing that, we use the obtained characteristic frequency (time) in the Drude formula, in order to obtain the resistivity of the system. The Hall mobility will also be studied by using the above results.

Gaona and Silva [6], inspired in the Gorter-Casimir (GC) two-fluid model of superconductivity [7], as well in the formalism used to treat the phase transitions in hydrogen-bonded ferroelectrics [8], have proposed an effective transverse Ising model (TIM) to describe both metallic and HTS superconductivity. We think that an extension of this previous model could be useful to describe some basic features of Beschoten et al [5] experiment.

Let us first make some considerations about two well-established phenomenological theories of the superconductivity. One of the main characteristics of the superconducting state is the macroscopic phase coherence of the wave function. In 1950, Ginzburg and Landau [9] introduced their phenomenological theory of superconductivity, where the order parameter must be identified with the macroscopic wave function  $\Psi$ . They started from the idea that  $\Psi$  represents some effective wave function of the superconducting electrons.  $\Psi$  can be normalized in such a way that  $|\Psi|^2$  is equal to the concentration  $n_s$  of superconducting electrons. It seems that the pioneering theory to treat the thermodynamic behavior of the superconducting state was proposed by Gorter and Casimir (GC) [7]. The two-fluid model of Gorter and Casimir was proposed to provide a basis for understanding the thermodynamic of the superconducting state. In this phenomenological theory it is assumed that the electrons in a superconductor are divided into two interpenetrating gases: the superconducting electrons, which have zero entropy, and the normal electrons, which have the usual properties of electrons in a normal metal. The normal electrons give up the condensation energy  $\beta$  when they become superconducting. The authors proposed that below  $T_c$  a fraction  $y$ , where  $y$  depends on the temperature  $T$  of the conduction electrons occupy a set of lower "condensed" energy states which are associated with the superconducting properties, while the fraction  $(1 - y)$  remains uncondensed. The free energy function for the whole electronic assembly is made up of two terms: one,  $F_N$ , is similar to the electronic free energy of a normal metal and the other,  $F_S$ , corresponds to the condensation energy associated with the condensed state ,

$$F_N = -\frac{1}{2}\gamma T^2, F_S = -\beta \quad (1)$$

where  $\gamma$  is the normal electronic specific heat coefficient, and  $\beta$  is a constant representing the condensation energy. Gorter and Casimir assumed that the phases were not independent, and proposed the following free energy function:

$$F(T, y) = (1 - y)^{1/2} F_N + y F_S . \quad (2)$$

Close to  $T_c$  we may express (2) in the standard Ginzburg-Landau form by expanding  $F(T, y)$  as a power series in  $y$ , assuming  $y = |\Psi|^2$ , and retaining the first nonvanishing terms. The model has acquired support by its success to interpret all properties of superconductors. In particular, the temperature dependence of the penetration depth is correctly predicted [10,11].

An inspection of the GC free energy relation (2) reveals that it contains two contributions: the first one is related to the property of free electrons in metals to transpose the Fermi barrier, being responsible for the transport properties and consistent with the Pauli exclusion principle. This term also contains a multiplicative factor which takes into account the fact that, below  $T_c$ , the number of normal electrons in the superconducting sample is a temperature dependent quantity. The second term of the GC free energy is responsible for the pairing of wave functions of the

electron condensate. This term is also temperature dependent (indeed, it is proportional to the number of the electrons in the condensate) and above  $T_c$  it does not contribute to the GC free energy. So, the GC free energy clearly displays the competition between these two terms.

On the other hand, a very successful model to treat various cooperative phenomena is the Ising model. However, the Ising model suffers from a deficiency, namely, the impossibility to exhibit a proper dynamics. Perhaps the simplest extension of the Ising model with a proper dynamics is the transverse Ising model (TIM). The TIM has been used to describe phase transition in ferroelectrics, ferromagnets and cooperative Jahn Teller systems [12]. This model Hamiltonian was first proposed by De Gennes [13] based on arguments developed by Blinc [14] to represent the basic features of hydrogen-bonded ferroelectrics of the  $\text{KH}_2\text{PO}_4$  family. In these systems the Ising term corresponds to the interaction between the protons at different lattice sites and the transverse field accounts for the possibility of protons occupying one of the two minima of a double potential well in a given site.

## 2. The Model Hamiltonian

Taking into account the preceding considerations, we propose the following effective Hamiltonian as a means to describe some transport properties of the  $\text{Bi}_2\text{Sr}_2(\text{Ca}_z\text{Pr}_{1-z})\text{Cu}_2\text{O}_{8+y}$ . We write:

$$H = -\Delta \frac{1}{2} \sum_i C_i - \Omega \sum_i \frac{1 - C_i}{2} X_i - \frac{1}{2} \sum_{ij} L_{ij} \frac{1 - C_i}{2} Z_i \frac{1 - C_j}{2} Z_j. \quad (3)$$

The main ideas that we have in mind are:

i-) The third term of (3) is an Ising-like term, where the operator  $Z$  is related to the wave function of the condensate electrons (holes), and the coupling  $L_{ij}$  favors the pairing of electrons (holes) at two different lattice sites, contributing to the coherence of their wave functions below the critical temperature  $T_c$ .

ii-) In the second term of (3) the operator  $X$  is related to the wave function of the normal electrons (holes), where  $\Omega$  represents the transverse field or tunneling constant. This term, which gives dynamics to the model, could be thought as the kinetic contribution to the model Hamiltonian. It could also represent a mimic for the motion of "free" electrons (holes) through the Fermi barrier, in close analogy with the tunneling of protons through the double-well potential barrier in the  $\text{KH}_2\text{PO}_4$ -like (hydrogen-bonded ferroelectrics) case (see [8]). However, while in  $\text{KH}_2\text{PO}_4$  the proton tunneling occurs between the right and left sides of the potential well, in the present model we consider the possibility of the electron (hole) to tunnel between the two discrete values of the phase  $\varphi$  of its wave function, namely between  $\varphi = 0$  and  $\varphi = \pi$ . According to this point of view, the spontaneous symmetry breaking will occur when the system makes the choice between one of these two particular phases of the wave function.

iii-) In the first term of (3),  $\Delta$  represents the metal-insulator activation energy, and the operator  $C$  will assume the eigenvalues  $+1$ , when the electron (hole) occupies the ground state (insulator), and  $-1$ , when it occupies the excited state (metal or superconductor). We observe that the electron (hole) could change the phase of its wave function only in the  $-1$  eigenstate of  $C$ . The interaction of electrons (holes) between different lattice sites, which leads to the cooperative effect of the superconductivity, occurs only in this last case.

With respect to the operators which appear in the Hamiltonian (3), we would like to make the following remarks:  $X$  and  $Z$  will be treated as linear independent operators in the space of spin-1/2, described by the Pauli matrices. The operator  $C$ , which has eigenvalues  $+1$  and  $-1$ , could be treated as a "z component" of a spin-1/2 operator in another space. In this way, the Hamiltonian (3) is defined in the space product of the operators  $C$  and  $(X, Y, Z)$ .

Before finishing this section, it is worth to mention that a Hamiltonian of the kind described by (3) was used by Ohtomi and Nakano [15] to explain the role of the pressure in determining the order of the phase transition in hydrogen-bonded ferroelectrics. They however treated (3) only in the static case, namely in the mean field approximation (MFA).

### 3. The Dynamics of the System

The Dynamics of the system will be treated by applying the RPA to the Heisemberg equations of motion. We have (with  $\hbar = 1$ )

$$\frac{d\langle P_i \rangle}{dt} = -i\langle [P_i, H] \rangle_t, \quad (4)$$

where the operators  $P_i = X_i, Y_i, Z_i$  must satisfy the Pauli commutation relations,

$$[X_i, Y_i] = i\delta_{ij}Z_i, \dots \quad (5)$$

To solve (4) in the RPA, we write

$$\langle P_i \rangle_t = \langle P_i \rangle + \delta \langle P_i \rangle e^{i\omega t}, \quad (6)$$

where in (6) we make the replacement of  $\langle P_i \rangle_t$  by a constant part  $\langle P_i \rangle$ , which is just the mean field approximation (MFA) expected value, plus a small time-dependent deviation  $\delta \langle P_i \rangle e^{i\omega t}$ .

Using (5) and (6), keeping only terms which are linear in the deviation  $\delta \langle P_i \rangle$  and by putting  $\langle Y \rangle = 0$ , we have the following set of linearized equations:

$$\begin{aligned} i\omega \delta \langle X \rangle - L_0 \left( \frac{1 - \langle C \rangle}{2} \right) \langle Z \rangle \delta \langle Y \rangle &= 0 \\ i\omega \delta \langle Y \rangle - \left[ \Omega - L_0 \left( \frac{1 - \langle C \rangle}{2} \right) \langle X \rangle \right] \delta \langle Z \rangle + L_0 \left( \frac{1 - \langle C \rangle}{2} \right) \langle Z \rangle \delta \langle X \rangle &= 0 \\ i\omega \delta \langle Z \rangle + \Omega \delta \langle Y \rangle &= 0 \end{aligned} \quad (7)$$

The solutions of (7) are:

$$\omega_1 = 0 \quad (8)$$

and

$$\omega_{2,3}^2 = \Omega \left[ \Omega - L_0 \left( \frac{1 - \langle C \rangle}{2} \right) \langle X \rangle \right] + \left[ L_0 \left( \frac{1 - \langle C \rangle}{2} \right) \langle Z \rangle \right]^2, \quad (9)$$

which are constrained by the zero-order solution

$$\left[ \Omega - L_0 \left( \frac{1 - \langle C \rangle}{2} \right) \langle X \rangle \right] \langle Z \rangle = 0. \quad (10)$$

In the following we are interested only in the characteristic frequency of the electrons (hole) in the non-superconducting phase, i.e., in the phase given by  $\langle Z \rangle = 0$ .

Putting  $\langle Z \rangle = 0$  in  $\omega_{2,3}^2$  given by (9) and dropping the subscripts for sake of simplicity we have:

$$\omega^2 = \Omega \left[ \Omega - L_0 \left( \frac{1 - \langle C \rangle}{2} \right) \langle X \rangle \right] \quad (11)$$

which leads to:

$$\omega_{\pm} = \pm \left\{ \Omega \left[ \Omega - L_0 \left( \frac{1 - \langle C \rangle}{2} \right) \langle X \rangle \right] \right\}^{1/2}. \quad (12)$$

In (12) the values of  $\langle C \rangle$  and  $\langle X \rangle$ , in the MFA, are given by

$$\langle X \rangle = \frac{1}{2} \tanh \left( \frac{\Omega}{2k_{\beta}T} \right) \quad (13)$$

and

$$\langle C \rangle = \tanh \left( \frac{\Delta - \Omega \langle X \rangle}{2k_{\beta}T} \right). \quad (14)$$

#### 4. Transport Properties

##### 1. Evaluation of the Resistivity

To evaluate the resistivity we will work with the Drude formula,

$$\rho = \frac{m}{pe^2\tau} \quad (15)$$

where  $m$  is the effective mass of the carriers,  $p$  their concentration,  $e$  the electronic charge and  $\tau$  the scattering time.

Let us now make the link between the dynamics of our model and the Drude formula. Since the carriers concentration must be an increasing function of the  $Ca$  concentration,  $f(z)$ , and must be a maximum when all the electrons (holes) occupies the excited state ( $\langle C \rangle = -1$ ) and a minimum for the ground state ( $\langle C \rangle = +1$ ), we propose:

$$p = n f(z) \left( \frac{1 - \langle C \rangle}{2} \right)^2, \quad (16)$$

where  $n$  is the density of carriers which are able to form cooper pairs. We also assume [16] that

$$n = \frac{2}{\xi_{ab}^2 \xi_c}, \quad (17)$$

where  $\xi_{ab}$  and  $\xi_c$  are respectively the in-plane and the perpendicular coherence lengths. The number 2 corresponds to the assumption that in the superconducting phase we will have only a cooper pair by "unit cell" of volume  $\xi_{ab}^2 \xi_c$ .

By putting

$$\tau = \frac{1}{2|\omega_{\pm}|} \quad (18)$$

in (15) we get

$$\rho = \left( \frac{2m}{e^2} \right) \frac{|\omega_{\pm}|}{p}, \quad (19)$$

and putting (12) and (16) into (19) we get:

$$\rho = \left( \frac{8m}{e^2 \hbar} \right) \frac{\left\{ \Omega \left[ \Omega - L_0 \left( \frac{1-\langle C \rangle}{2} \right) \langle X \rangle \right] \right\}^{1/2}}{nf(z) (1 - \langle c \rangle)^2}. \quad (20)$$

One immediate consequence of relation (20) is that the onset of the superconductivity is reached when the term in the square-root goes to zero. This corresponds to the softening of the mode which frequency is given by equation (12).

## 2. Hall Mobility

To make a connection between our model and the Hall mobility  $\mu$ , we will consider the one-band model [17] where the Hall effect is inverse proportional to the carrier concentration  $p$ :

$$R_H = \frac{1}{pe}. \quad (21)$$

Using this assumption we will have for the mobility

$$\mu = \frac{R_H(T, z)}{\rho(T, z)} = \frac{e}{2m |\omega_{\pm}|}, \quad (22)$$

where we have used (19) to obtain the right side of (22).

Putting (12) into (22) we finally obtain

$$\mu = \frac{|e| \hbar}{2m} \left\{ \Omega \left[ \Omega - L_0 \left( \frac{1 - \langle C \rangle}{2} \right) \langle X \rangle \right] \right\}^{-1/2}. \quad (23)$$

## 3. A Possible Mode of Vibration

We also can evaluate the resistivity with an equivalent way of writing the Drude formula

$$\rho = \frac{mv_F}{pe^2 l}. \quad (24)$$

where  $v_F$  is the Fermi velocity and  $l$  the mean free path of the electrons (holes).

As a means to determine the electron (hole) mean free path we start with the classical formula

$$l = \frac{1}{n\pi r^2}. \quad (25)$$

We observe that in (25) we have considered the number of scatter centers by unit of volume to be equal to the density of Cooper pairs given by (17). We can justify this by the argument that the electric conduction always occurs in a regime of charge neutrality.

The electron (hole) mean free path,  $l$ , remains to be linked to the model's dynamics. Alternatively, we can work with the collision cross-section  $\pi r^2$  taking into account equation (25). To do this let us write the "resonance" condition:

$$\frac{1}{2}K\overline{r^2} = 2\hbar|\omega_{\pm}|. \quad (26)$$

In (26), we have considered the equality in energy between a classical harmonic oscillator of elastic constant  $K$  and the energy separation of a two-level quantum system, which frequencies  $\omega_{\pm}$  are given by (12).

Initially we are going to determine the elastic constant  $K$  through the following arguments. It seems natural to consider that the Debye frequency  $\omega_D$  and the mass  $M$  of the oxygen ion must play an important role in the scattering of the charge carriers. So, let us write the relation:

$$K = \alpha M \omega_D^2. \quad (27)$$

We can suppose that in (27)  $K$  is an effective elastic constant, where  $\alpha$ , an adjustable perhaps small pure number, could be interpreted as a mimic for a weak coupling regime of the BCS [18] model.

Inserting the informations contained in the equations (25) and (26) into (24), we obtain

$$\rho = \left( \frac{4\pi n \hbar m v_F}{e^2 K} \right) \frac{|\omega_{\pm}|}{p}, \quad (28)$$

and putting (12) and (16) into (28) we get:

$$\rho = \left( \frac{16\pi m v_F}{e^2 K} \right) \frac{\left\{ \Omega \left[ \Omega - L_0 \left( \frac{1-\langle C \rangle}{2} \right) \langle X \rangle \right] \right\}^{1/2}}{f(z) (1 - \langle C \rangle)^2}. \quad (29)$$

Since (19) and (28) are two equivalent ways of writing the resistivity, we obtain

$$K = 2\pi n \hbar v_F. \quad (30)$$

On the other hand an estimate of the Fermi velocity  $v_F$  was given in reference [19], in terms of the electron (hole) mass and the in-plane coherence  $\xi_{ab}$ , namely,

$$v_F = \frac{3}{2} \frac{\hbar}{m \xi_{ab}}. \quad (31)$$

Putting (17), (27) and (31) into (30), we get

$$K = \alpha (M \omega_D^2) = \frac{6\pi \hbar^2}{\xi_{ab}^3 \xi_c m}. \quad (32)$$

The fact that the right side of (32) depends only on the in-plane and perpendicular coherence lengths and on the effective mass  $m$  of the carriers leads to the conjecture that it is possible to find a mechanism accounting for the elastic constant  $K$ . In the following we are going to look for this possibility.

In a previous work by one of the present authors [19], an effective potential was proposed to describe the interaction between electron pairs (holes) relating the size of this super carrier to the energy of the bound state. By considering the particular situation that the particle occupies the minimum of this effective potential and that the motion of the pair is confined in a plane [20], we found that this problem could be mapped into that of the quantum mechanical behavior of a particle described by polar coordinates in a plane. The solution of this quantum mechanical problem leads to an eingevale spectrum which is doubly degenerate, except for the ground state. In the first excited state we have two eingevales of the momentum  $q$ , namely:

$$q_{\pm} = \pm \frac{\hbar}{\xi_{ab}} . \quad (33)$$

We may consider that a pair of electrons (holes) circling in opposite directions establishes a ring of charge and that an oxygen ion could experiment the electric field of these pair of carriers. This leads to an oscillatory motion of the oxygen ion perpendicular to the plane, and in the case of small oscillations, this motion could be described by a harmonic oscillator which frequency  $\omega_0$  is given by:

$$M\omega_0^2 = \frac{e^2}{\pi\epsilon_0\xi_{ab}^3} . \quad (34)$$

Now, if we make the identification of the left side of (34) with the elastic constant  $K$  given by (32), we obtain:

$$\xi_c = \left( \frac{4\pi\epsilon_0}{e^2} \right) \frac{3\pi}{2} \frac{\hbar^2}{m} . \quad (35)$$

So, according to our model the perpendicular coherence length  $\xi_c$  is inversely proportional to the effective mass  $m$  of the charge carriers.

## 5. Comparison with the Experiments and Numerical Estimates

Let us now compare the experimental results of Beschoten et al [5] with the present model. We may consider that the role played by the addition of  $Pr$  in the  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$  is to promote certain kind of dilution which is responsible by the decreasing of the critical temperature of the material. We assume that the dilution affects both the values of the "tunneling constant"  $\Omega$ , as well as the activation energy  $\Delta$ . In order to understand this, let us look at figure 1. In this figure we have a double-well potential where the minimum corresponds to one of the two values of the discrete phases of the local wave function,  $\varphi = 0$  or  $\varphi = \pi$ . The parameter  $\Delta$  corresponds to the separation in energy between the ground and the excited states. In figure 2 we show that the effect of the tunneling is to make the splitting of the excited state in two sub-levels. The system could become superconductor as far as  $\Omega/2 > \Delta$ . Therefore the critical concentration of  $Ca$  ions ( $z_c$ ), i.e., the concentration below which we have no more the superconducting state, will occur at  $\Omega = 2\Delta$ .

Looking at the resistivity formula given by (20), we can observe that its temperature dependence is controlled by the parameters of the model. So for  $\Omega > 2\Delta$ , the onset of the superconductivity is reached when:

$$\langle X \rangle_{T=T_c} = \frac{2\Omega}{L_0 (1 - \langle C \rangle_{T_c})} , \quad (36)$$

where  $\langle X \rangle$  and  $\langle C \rangle$  are given by (13) and (14), respectively.

In the numerical estimates of the resistivity (equation (20)), we have three adjustable parameters: the tunneling constant  $\Omega$ , the activation energy  $\Delta$  and the ratio between the coupling and the tunneling constants  $L_0/\Omega$ . Table 1 shows the values of the parameters adjusted to reproduce the experimental data findings of Beschoten et al [5]. In figure 3 we plot in a semi-logarithmic scale the resistivity (equation (20)) of the  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$  as a function of the temperature for various values of the dilution  $z$ . We used, in evaluate  $n$  given by (17),  $\xi_{ab} = 15\text{\AA}$  and  $\xi_c = 3\text{\AA}$  [21], and  $m$  is the electron rest mass.

| $z$ | $\Delta$ | $\Omega$ | $L_0/\Omega$ |
|-----|----------|----------|--------------|
| 0.3 | 12.62    | 24.0     | 4.2          |
| 0.4 | 9.68     | 19.0     | 4.0          |
| 0.5 | 7.61     | 15.0     | 3.8          |
| 0.6 | 5.35     | 11.0     | 3.2          |
| 0.7 | 1.31     | 2.8      | 4.1          |
| 0.9 | 0.9      | 2.1      | 5.0          |

Table 1: Adjusted parameters values for various  $Ca$  concentrations ( $z$ ) in  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$  ( $\Delta$  and  $\Omega$  are in units of  $10^{-21}J$ ).

A linear relationship between  $z$  and  $\Delta/\Omega$  can be inferred from the data of table 1. From this relation we can estimate a value for the critical concentration,  $z_c$ , approximately 0.51, corresponding to  $\Delta/\Omega = 1/2$ .

In the numerical estimates of the resistivity for the  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$ , in the version of equation (29), we have used  $\alpha$  as an adjustable parameter.  $M$  is the mass of the oxygen ion,  $\omega_D$  is the Debye frequency given by  $k_B\theta_D/\hbar$ , where we used for the Debye temperature  $\theta_D = 300K$  [22]. We have adjusted  $\alpha = 0.005$ , consistent with the value of  $M\omega_0^2$  obtained in (34).

We also obtain for  $\xi_c$ , using equation (35), the value of approximately  $2.5\text{\AA}$ .

Beschoten et al [5] have investigated the inverse of the Hall mobility,  $\mu^{-1}$ , in  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$  cuprate. They found that the mobility of the conduction holes evolves from the insulator ( $z < z_c = 0.52$ ) to the high temperature superconductor ( $z > z_c$ ) by using a one-band model with a single scattering time in order to fit the experimental data. Their main result is that, even for high Ca content,  $z > z_c$ , where the system becomes a superconductor, they still clearly observe a minimum in the inverse mobility, although its resistivity is purely metallic (this indicates that, even in these samples, the superconducting wave function is formed by states which are spatially localized).

However in our results we can not observe those minima in the inverse mobility (equation (23)) for  $z > z_c$ , although they are present for  $z < z_c$ .

In figure 4 we use the zeros of the inverse Hall mobility and the minimum of it (for  $z < z_c$ ) as a mean to construct a diagram separating the various regimes: insulating, superconducting and metallic. This diagram agree with the experiments (figure 4 of reference [5]) unless for the fact that it does not reproduce the coexistence of superconducting and localization (insulating).

## 6. Concluding remarks

It seems that the model proposed in this paper can capture the main features which are present in the transport properties of the  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$ . In particular, we have evaluated the resistivity which agree with the experimental data of Beschoten et al [5].

From the evaluated inverse Hall mobility we extracted a diagram  $T$  (temperature) versus  $z$  (concentration of  $Ca$  ions) showing the various regimes of the system, namely metallic, localized (insulating) and superconducting regions.

By comparing two alternative ways of writing the Drude formula of resistivity, and by finding a possible mode of vibration (equation (34)) very relevant to the problem, we can lead to a relation which links the effective mass of the carrier (hole) to the perpendicular correlation length  $\xi_c$  (equation (35)).

The present research has been supported partly by CNPq-Brazil.

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FIG. 1. Figure 1: The double-well potential, showing the ground and first excited states. At the minimal, the local phase of the wave function is  $\varphi = 0$  or  $\varphi = \pi$ .



FIG. 2. Figure 2: Splitting of excited state of the potential well, due to the tunneling term  $\Omega$ . When  $\Omega/2 \geq \Delta$  the system is in the superconductor regime (for  $z > z_c$ ).



FIG. 3. Figure 3: Logarithm of the resistivity as a function of the temperature, for various values of  $Ca$  concentration ( $z$ ). Theoretical results of this work (solid lines) are compared with the experimental ones (circles) of Beschoten et al [5].



FIG. 4. Figure 4: Inverse mobility minima corresponding temperature  $T_{\min}$  (open squares) and superconducting transition temperature  $T_c$  (filled squares) for  $Bi_2Sr_2(Ca_zPr_{1-z})Cu_2O_{8+y}$ .